C1 Graph sketching 1 – cubics and reciprocals

Edexcel Examination Board (UK)

Book used with this handout is Heinemann Modular Mathematics for Edexcel AS and A-Level, Core Mathematics 1 (2004 edition).

Snezana Lawrence © 2007
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**Sketching cubics**

To sketch a curve, calculate its roots first of all, then calculate few points around the roots. This is not a very accurate method, but we are just beginning!

Sketch the following curves

1. \( y = (x - 5)(x - 2)(x - 1) \)

2. \( y = (x + 5)(x + 1)(x - 1) \)
3. \( y = x(x + 2)(x - 2) \)

4. \( y = x^3 - 3x^2 - 4x \)

Now do the exercise 4A on page 42.
Sketch briefly the curves with equations

1. \( y = x^3 \)

2. \( y = -x^3 \)

3. \( y = (x-1)^3 \)

4. \( y = (1-x)^3 \)

Exercise 4B page 45.
Historical background to quadratic, cubic, and quartic equations

When we say that Babylonians for example, or Egyptians, knew about quadratic or cubic equations and how to solve them, we mean that they used certain prescribed methods to solve problems that we would today write, using algebra, as such equations.

Euclid in his Elements (c. 300) developed a geometrical approach to solving quadratics in Proposition 11, Book II. Propositions VI-28 and VI-29 considered equations such as

\[ x^2 - ax + b^2 = 0 \quad \text{and} \quad x^2 - ax - b^2 = 0 \]

where \( a \) and \( b \) represented the lengths of line segments.

The great advancements to the solutions of quadratic and cubic equations came through two Italian mathematicians from the 15th century: Luca Pacioli (1445-1517) and Scipione del Ferro (1465-1526). Whilst Luca published his results in one of the famous books on mathematics Summa de arithmetica, geometria, proportioni et proportionalita (Venice 1494), in which he wrote pretty much all that was known of mathematics in Italy at the time, del Ferro left his work unpublished in a manuscript which was later adopted by some of his friends who were interested in mathematics.
Reciprocal curves

You will now meet with the reciprocal curves. They are reciprocals to the usual curves you studied so far (\(y = x\)), and are of the form \(y = \frac{k}{x}\), where \(k\) is a constant number.

Using a table of values sketch the curve \(y = \frac{1}{x}\)

<table>
<thead>
<tr>
<th>(x)</th>
<th>-10</th>
<th>-8</th>
<th>-6</th>
<th>-4</th>
<th>-2</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>(y)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>

![Graph of the reciprocal curve](image-url)
Reciprocal curves of this kind fall into two categories: when the constant number $k > 0$ and when $k < 0$.

$k > 0$

$$f(x) = \frac{2}{x}$$

$k < 0$

$$f(x) = \frac{-2}{x}$$
Sketch on the same diagram:

1. \( y = \frac{4}{x} \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>-10</th>
<th>-8</th>
<th>-6</th>
<th>-4</th>
<th>-2</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td></td>
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</tr>
</tbody>
</table>

and \( y = \frac{7}{x} \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>-10</th>
<th>-7</th>
<th>-6</th>
<th>-4</th>
<th>-2</th>
<th>2</th>
<th>4</th>
<th>7</th>
<th>8</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td></td>
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</tr>
</tbody>
</table>
2. \( y = \frac{-3}{x} \)

\[
\begin{array}{cccccccccc}
  x & -10 & -8 & -6 & -3 & -2 & 2 & 3 & 6 & 8 & 10 \\
  y & & & & & & & & & & \\
\end{array}
\]

and \( y = \frac{-5}{x} \)

\[
\begin{array}{cccccccccc}
  x & -10 & -8 & -5 & -4 & -2 & 2 & 5 & 6 & 8 & 10 \\
  y & & & & & & & & & & \\
\end{array}
\]
Answers and notes

Write your own – and send me an email if I’ve made a mistake/you found a misprint anywhere in this booklet... 😊