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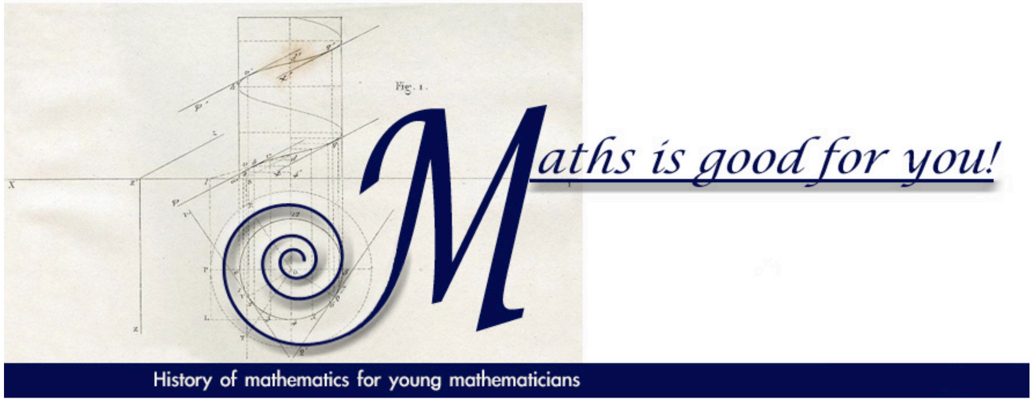
Worksheet Fibonacci3

Teacher

Student

Class



Golden Ratio, number ϕ, and Fibonacci's numbers

Fibonacci's numbers have a curious property: if you divide them in succession with the previous numbers in the sequence, the result approximates closer and closer to the number ϕ = 1.6180339887... Fibonacci Number sequence is a sequence which looks like this:

1, 1, 2, 3, 5, 8, 13, 21, 34, 55...

Divide the number from this sequence by preceding number, and see what happens:

✍ 2 ÷ 1 = 2   3 ÷ 2 =   5 ÷ 3 =   8 ÷ 5 =   13 ÷ 8 =   21 ÷ 13 =

You can investigate this further by doing the Fibonacci's worksheet no. 2, or to learn more about Fibonacci himself, Fibonacci's worksheet no. 1. They can all be found on worksheets page at www.mathsisgoodforyou.com.

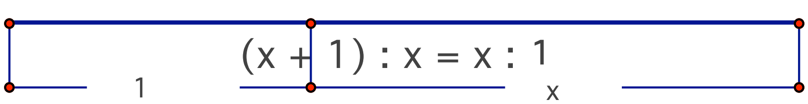
Number ϕ is an irrational number – and some say it is more irrational than other irrational numbers because it is used by artists – a lot! Its approximate value is 1.6180339887... but because it is irrational, it never ends, and doesn't have any reoccurring pattern of decimals.

Funny thing is – number ϕ is equal to the ratio that you can find in the Golden Section. Golden Section, or Ratio, or Mean, has been known to the ancient Greeks. In fact the number ϕ probably got its name from Phidias (in Greek ϕ would be Phi), a famous Athenian sculptor who often used the Golden Section proportions in his work. Golden Section was also described in Euclid's *Elements* in few places - and here is another little task for you – find all theorems that refer to it!

Here is how Golden Ratio is described by Euclid:

> *A straight line is said to have been cut in extreme and mean ratio when, as the whole line is to the greater segment, so is the greater to the less.*

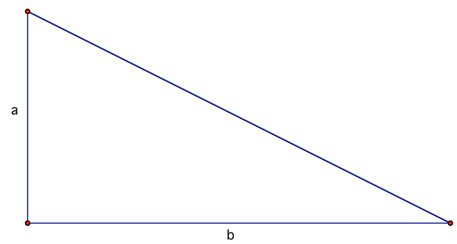
Let us think carefully what this means. You need to divide the line in such a way that the bigger part of it is to the whole line in the same ratio as the smaller is to the bigger part. Geometrically this should look like the following diagram:



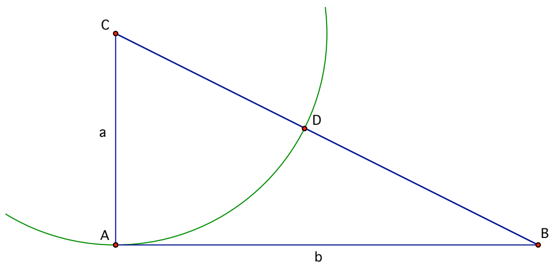
On the next page you will learn how to construct Golden Ratio and how you can use it to draw Golden Rectangle and the logarithmic spiral.

Geometrical construction of Golden Ratio

First draw a right angled triangle like the one below.

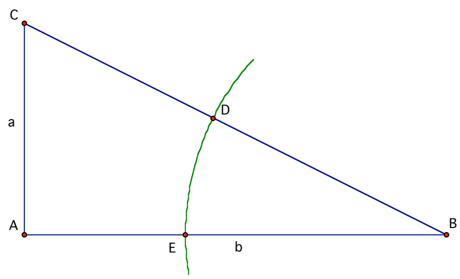


Make sure that the base of this triangle (side ***b***) is twice the size of the side ***a***.



Draw arc of a circle having a centre at vertex C and the radius which is the same size as the side ***a*** of the triangle ABC.

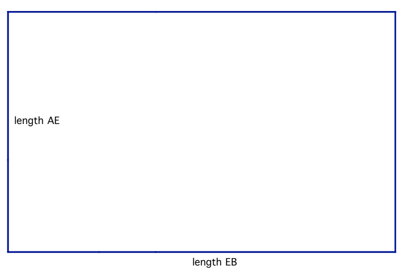
This arc will cut the hypotenuse of the triangle in a point which we will call D.



Now draw another arc – this time the centre should be the point B, and the radius should be of length equal to BD.

This arc will cut the line AB in point E. The point E marks the Golden Ratio – AB is to EB in same ratio as EB is to AE. Mathematically, we write that:

$$AB : EB = EB : AE$$



Using lengths AE and EB you can draw a rectangle having these lengths for its sides.

Make a series of squares in this rectangle starting from one of its sides like this

Finally you can make arcs in each of these squares which all join up – this will make a logarithmic spiral.

Beautiful isn't it?!