Möbius strip

As mentioned on the web page which introduced us to Möbius strip, investigating and studying properties of it belongs to a field of mathematics which is called topology.

Topology is concerned by the classification of surfaces and by looking at their common and distinguishing properties. We will investigate Möbius strip by using topological approach.

First consider cylinder. It is a commonly known solid, which you can make yourself by cutting a rectangle from a piece of paper and joining the corresponding sides. In other words, the vertices of your rectangle that are up remain up and the vertices of your rectangle that are down remain down and the two corresponding edges are just joined (use tape or glue).

Cylinder has two surfaces - inside and outside.

Now consider using the same strip, but joining the top of the left-hand side edge with the bottom of the right-hand edge.

You will get Möbius strip.

Does Möbius strip have two surfaces? Try to explain.

August Ferdinand Möbius

the inventor of Möbius strip was born on 17 Nov 1790 in Schulpforta, Saxony (now Germany) and died on 26 Sept 1868 in Leipzig, Germany.

When he was 19, he graduated from his college and went to study at the University of Leipzig. His family wanted him to study law so he started the course in law, but soon discovered that his real interest was in mathematics. He changed the course the next year.

Möbius also studied astronomy. His teacher in Göttingen was the famous mathematician Gauss.

Although the object that is the matter of investigation in this worksheet we now know as a Möbius strip, it was not Möbius who first described this object, but his colleague Listing.
**Möbius band in other dimensions?**

If you think of two dimensional strip (rectangle) from which you started as a model which you can apply in other dimensions, than you can do the same thing with a torus. It would be much harder to make; so here are some images to help you imagine the process. First you have a kind of tube which you just use as it is and join the two ends to get a torus.

But if you use the same principle that you did for making Möbius strip to a tube, you will get a Klein bottle. Klein bottle doesn’t have inside nor out - they are the same. If you manage to make one, try putting some liquid into it and you’ll see!

**Investigation**

Cut your Möbius Band down the middle. What is the result? Write a description of what happens.

Next cut your new strip down the middle again. What is the result? Explain.

Make another Möbius Band. Cut it not down the middle but one third from the edge. What is the result? Explain and draw some conclusions.

Form another strip of paper, and this time glue the ends together with a twist of 540 (three times what you did originally to create your first Möbius strip) degrees. Again we have a surface with one edge and one side. It also is a Möbius Band, but the way it is put into our three dimensional space is different from the previous one.

This is an important change from the original Möbius strip. Try to explain the difference between the two in terms of
- the objects themselves
- the way they are part of a space.

Cut this new object down the middle. What happens? Explain.

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